Journal of Hydrology 380 (2010) 121–134

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

A copula-based joint deficit index for droughts

Shih-Chieh Kao [a], Rao S. Govindaraju [b,*]

[a] Computationl Sciences and Engineering Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA
[b] School of Civil Engineering, Purdue University, West Lafayette, IN 47907, USA

ARTICLE INFO

Article history:
Received 29 January 2009
Received in revised form 16 June 2009
Accepted 27 October 2009

This manuscript was handled by K. Georgakakos, Editor-in-Chief, with the assistance of Andreas Bardossy, Associate Editor

Keywords:
Copulas
Drought
Standardized precipitation index
Frequency analysis
Risk assessment

SUMMARY

Current drought information is based on indices that do not capture the joint behaviors of hydrologic variables. To address this limitation, the potential of copulas in characterizing droughts from multiple variables is explored in this study. Starting from the standardized index (SI) algorithm, a modified index accounting for seasonality is proposed for precipitation and streamflow marginals. Utilizing Indiana stations with long-term observations (a minimum of 80 years for precipitation and 50 years for streamflow), the dependence structures of precipitation and streamflow marginals with various window sizes from 1- to 12-months are constructed from empirical copulas. A joint deficit index (JDI) is defined by using the distribution function of copulas. This index provides a probability-based description of the overall drought status. Not only is the proposed JDI able to reflect both emerging and prolonged droughts in a timely manner, it also allows a month-by-month drought assessment such that the required amount of precipitation for achieving normal conditions in future can be computed. The use of JDI is generalizable to other hydrologic variables as evidenced by similar drought severities gleaned from JDIs constructed separately from precipitation and streamflow data. JDI further allows the construction of an inter-variable drought index, where the entire dependence structure of precipitation and streamflow marginals is preserved.

Introduction

Drought, as a prolonged status of water deficit, has been a challenging topic in water resources management. It is perceived as one of the most expensive and least understood natural disasters. In monetary terms, a typical drought costs American farmers and businesses \$6–8 billion each year (WGA, 2004), more than damages incurred from floods and hurricanes. The consequences tend to be more severe in areas such as the mid-western part of the United States, where agriculture is the major economic driver. Unfortunately, though there is a strong need to develop an algorithm for characterizing and predicting droughts, it cannot be achieved easily either through physical or statistical analyses. The main obstacles are identification of complex drought-causing mechanisms, and lack of a precise (universal) scientific definition for droughts.

When a drought event occurs, moisture deficits are observed in many hydrologic variables, such as precipitation, streamflow, soil moisture, snow pack, ground water levels, and reservoir storage. Focusing on various types of deficits, droughts are categorized differently. For example, meteorological droughts are based on deficits in precipitation, agricultural droughts on deficits in soil moisture, and hydrologic droughts on streamflow deficits (Dracup et al., 1980). Though these different types of deficits tend to be positively correlated and are likely responding to the same trigger, they exhibit diverse temporal and spatial scales. An overall drought indicator, that encompasses multiple types of deficits and relevant temporal scales, is therefore difficult to produce owing to the complicated dependencies in the variables that are used to characterize droughts.

Given their somewhat nebulous nature, the status of droughts is often assessed by various indices that are derived from hydrologic variables. Some early indices include: Munger's Index (Munger, 1916), Blumenstock's Index (Blumenstock, 1942), and Antecedent Precipitation Index (McQuigg, 1954). Palmer (1965) proposed a moisture index (Palmer Drought Severity Index, PDSI) based on water budget accounting using precipitation and temperature data. PDSI soon became a popular choice for drought assessment and is widely used even today (Dalezios et al., 2000; Kim et al., 2003). One reason for its success is that it provides an opportunity to assess droughts using multiple sources of observations (precipitation and temperature). Nevertheless, PDSI has several limitations (see Alley, 1984; Guttman, 1991, 1998; Guttman et al., 1992). For instance, the soundness of proposed water balance model is questioned, the temporal scale of PDSI is not clear, and the values of PDSI possess neither a physical (such as required rainfall depth) nor statistical meaning (such as recurrence probability).

* Corresponding author. Tel.: +1 765 496 3402; fax: +1 765 496 1988.
E-mail addresses: kaos@ornl.gov (S.-C. Kao), govind@purdue.edu (R.S. Govindaraju).

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doi:10.1016/j.jhydrol.2009.10.029

Some other commonly used indices include: Crop Moisture Index (CMI; Palmer, 1968), Surface Water Supply Index (SWSI; Shafer and Dezman, 1982), Vegetation Condition Index (VCI; Kogan, 1995), and Climate Prediction Center (CPC) Soil Moisture Index (Huang et al., 1996). Another popular index – Standardized Precipitation Index (SPI) – was introduced by McKee et al. (1993). Based on a given window size, the rainfall depth is transformed to its corresponding cumulative probability, and then mapped onto the standard normal scale. The probabilistic nature of SPI allows it to be comparable among various locations and variables, and it can be further interpreted in terms of recurrence intervals (or return periods). While SPI is widely adopted as a general tool for drought assessment (e.g. Cancelliere et al., 2007), it can lead to confusion because inconsistent results may emerge under different window sizes (unfortunately, there is no representative window). Multiple SPIs with various temporal scales (e.g. 3-, 6-, 9-, 12-month) need to be examined together in order to form an overall judgment of a drought. Besides, SPI cannot account for seasonal variability, i.e. a given amount of precipitation should have different connotation in the rainfall season when compared to the dry season.

Another approach for drought analysis follows Yevjevich (1967), who applied statistical theory of runs for modeling drought events. Based on a given demand (for instance, mean discharge), the observed time series are divided into wet events (values greater than demand) and dry events (values less than demand). By analyzing properties of historical dry events (such as duration, severity, and magnitude), frequency of a drought is estimated. Many hydrologic drought analyses have followed this rationale (such as Dracup et al., 1980; Zelenhasić and Salvai, 1987; Frick et al., 1990; Kim et al., 2003; Abi-Zeid et al., 2004; Cancelliere and Salas, 2004; Salas et al., 2005). However, the determination of the threshold demand level relies on subjective assumptions, and it is possible that a small change in this threshold will dramatically alter the abstracted events (a long drought may be broken into several short droughts). Heim (2002) provided an extensive review of drought literature in the twentieth century.

The current consensus among researches is that no single approach is best for characterizing droughts. The drought status that is assessed from one indicator often does not correspond well with that obtained from a different indicator because of the complicated physical connections between infiltration, evapotranspiration, groundwater motion, base flow and direct runoff. In addition, droughts result from cumulative effects of water shortages over different periods of time. To successfully assess a drought, information from various sources need to be examined simultaneously. This is currently achieved by the US Drought Monitor where the severity of a drought (D0–D4, see Table 1 for the probabilities of occurrence and drought condition for each category drawn from Svoboda et al. (2002)) is based on various indicators (PDSI, CPC Soil Moisture, USGS weekly, Percent of normal, SPI, and VCI). Nevertheless, though an Objective Blend of Drought Indicators (OBDI, a linear weighted average of several indicators) is adopted as a reference of the overall severity, the decision of final drought status relies on subjective judgment of local personnel. With subjective intervention in specification of drought status, a rigorous analysis has not been feasible in physical and probabilistic characterization of droughts, and a scientifically-defensible analysis of droughts is still lacking. Since drought maps form the basis of subsidies received by states, it is all the more important that they be quantified in an objective fashion.

Table 1
The categories used in US Drought Monitor along with the corresponding SI values.

SI values	Probabilities of occurrence (%)	Drought condition	Drought monitor category
−0.84 to −0.52	20–30	Abnormally dry	D0
−1.28 to −0.84	10–20	Drought – moderate	D1
−1.64 to −1.28	5–10	Drought – severe	D2
−2.05 to −1.64	2–5	Drought – extreme	D3
<−2.05	<2	Drought – exceptional	D4

There is a need to develop objective standards for analyzing records and specifying drought status based on multiple variables. This will be achieved through the construction of a joint indicator that draws on information from multiple sources, and will therefore enable better estimation of drought return period, persistence of droughts at a given severity, future risk assessment, and improved identification of possible portents of droughts. So far, the major stumbling block to such an approach has been our inability to describe the complicated dependent relationships between various drought-related variables. To accomplish this goal, copulas are employed in this study to facilitate the identification and construction of dependence structure of droughts, and to further our understanding into the statistical nature of droughts and our ability to characterize them.

The flexibility offered by copulas for constructing joint distributions is evident from related studies on rainfall frequency analysis (De Michele and Salvadori, 2003; Grimaldi and Serinaldi, 2006b; Kao and Govindaraju, 2007a; Zhang and Singh, 2007; Kuhn et al., 2007), flood frequency analysis (Favre et al., 2004; De Michele et al., 2005; Shiau et al., 2006; Zhang and Singh, 2006; Renard and Lang, 2007), trivariate frequency analysis (Grimaldi and Serinaldi, 2006a; Salvadori and De Michele, 2006; Genest et al., 2007; Kao and Govindaraju, 2008), bivariate return periods (Salvadori and De Michele, 2004), groundwater parameters (Bárdossy, 2006), drought frequency analysis (Shiau, 2006), probabilistic structure of storm surface runoff (Kao and Govindaraju, 2007b), multivariate L-moment homogeneity test (Chebana and Ouarda, 2007), remote sensing data (Gebremichael and Krajewski, 2007), tail dependence (Poulin et al., 2007), rainfall IDF curves (Singh and Zhang, 2007), and sea storm analysis (De Michele et al., 2007). A review of copulas in Genest and Favre (2007) indicated that application of copulas in hydrology is still in its nascent stages, and their full potential for analyzing hydrologic problems is yet to be realized. The detailed theoretical background and descriptions for the use of copulas can be found in Nelsen (2006) and Salvadori et al. (2007).

We propose a new drought indicator using copulas that enables us to compute a probability-based overall water deficit index from multiple drought-related quantities (or indices). Precipitation and streamflow data from Indiana are adopted here for purposes of demonstration. By further invoking copulas, we construct joint distributions of droughts where marginal distributions of relevant variables and their dependence structures can be fully preserved. The joint distribution provides an objective description of the overall deficit status, and paves the path for computation of probabilistic quantities such as return period and associated risk of a drought.

The remainder of the paper is organized as follows. A brief introduction to copulas, along with some properties that are central to the proposed study are presented in "Mathematical formulation", along with the construction of drought marginals, dependence structure, and copula-based joint deficit index. A description of the selected precipitation and streamflow data from Indiana is provided in "Data used in this study". Results and applications of the proposed joint index, including comparisons between precipitation and streamflow deficits and drought potential assessment are described in "Results and applications", and finally conclusions are presented in "Conclusions".

Mathematical formulation

Over the last decade, copulas have emerged as a powerful approach in simplifying multivariate stochastic analysis. Sklar (1959) showed that for d-dimensional continuous random variables $\{X_1, \ldots, X_d\}$ with marginal cumulative distribution functions (CDFs) $u_j = F_{X_j}(x_j)$, $j = 1, \ldots, d$, there exists one unique d-copula $C_{U_1,\ldots,U_d}$ such that:

$$H_{X_1,\ldots,X_d}(x_1, \ldots, x_d) = C_{U_1,\ldots,U_d}(u_1, \ldots, u_d) \tag{1}$$

where u_j is the j^{th} marginal and $H_{X_1,\ldots,X_d}$ is the joint-CDF of $\{X_1, \ldots, X_d\}$.

Previous studies have indicated that copulas perform well for bivariate problems, and in particular, several families of Archimedean copulas, including Frank, Clayton, and Gumbel-Hougaard, have been popular choices for dependence models because of their simplicity and generation properties (Nelson, 2006). However, the direct extension of Archimedean copulas to higher orders (>2) is very limited because they impose severe restrictions on the pairwise mutual dependencies that can be accommodated, and compatibility conditions are far more difficult to satisfy in higher dimensional problems (Kao and Govindaraju, 2008).

Consequently, we employ empirical copulas that are rank-based empirical measures of joint cumulative probability (Nelsen, 2006). For sample size n, the d-dimensional empirical copula C_n is:

$$C_n\left(\frac{k_1}{n}, \frac{k_2}{n}, \ldots, \frac{k_d}{n}\right) = \frac{a}{n} \tag{2}$$

where a is the number of samples $\{x_1, \ldots, x_d\}$ with $x_1 \le x_{1(k_1)}, \ldots, x_d \le x_{d(k_d)}$, and $x_{1(k_1)}, \ldots, x_{d(k_d)}$ with $1 \leqslant k_1, \ldots, k_d \leqslant n$ are the rank statistics from the sample. When a sufficiently large sample size is available, empirical copulas can be used to construct non-parametric joint empirical probability distributions, which tend to be more computationally efficient. This is a desirable feature for drought analysis. The question of sampling deficiencies, especially for small data sets, may be accounted for using the methods described in Rüeschendorf (1985) and Drouet-Mari and Kotz (2001). As an example of parametric distribution, a brief description of Student t-copulas is provided in the appendix.

Kendall distribution function – K_C

For given d-variate sample marginals $\{u_{1x}, \ldots, u_{dx}\}$, a copula $C_{U_1,\ldots,U_d}(u_{1x}, \ldots, u_{dx})$ is the cumulative probability measure $P[U_1 \leqslant u_{1x}, \ldots, U_d \leqslant u_{dx}] = q$. There may exist other sample marginals $\{u_{1y}, \ldots, u_{dy}\}$ with the same value of cumulative probability $C_{U_1,\ldots,U_d}(u_{1y}, \ldots, u_{dy}) = q$. If this cumulative probability q is treated as an indicator, i.e. events with same value q are assumed to cause similar impact (e.g. in drought analysis it can be defined as joint deficit status, a smaller q implying overall drier conditions), then it will be of interest to know what the probability is for a random event $\{U_1, \ldots, U_d\}$ with $C_{U_1,\ldots,U_d}(U_1, \ldots, U_d) = q$, or $C_{U_1,\ldots,U_d}(U_1, \ldots, U_d) \le q$. In this context, Kendall distribution function K_C (Nelsen et al., 2003; Nelson, 2006) that is defined as the probability measure of the set $\{(U_1, \ldots, U_d) \in [0,1]^d |\ C_{U_1,\ldots,U_d}(U_1, \ldots, U_d) \le q\}$ is useful:

$$K_C(q) = P[C_{U_1,\ldots,U_d}(U_1, \ldots, U_d) \le q] \tag{3}$$

Salvadori and De Michele (2004) adopted K_C for defining a secondary return period for bivariate Archimedean copulas. The most appealing feature of K_C is that it can help project multivariate information onto a single dimension. An illustration of K_C for a bivariate case is shown in Fig. 1 using Gumbel-Hougaard Archimedean copula with dependence parameter $\theta = 2$ (see Nelsen, 2006). In Fig. 1, the curved lines represent level curves with $C(u, v) = q$, and K_C yields the probability measures of the shaded areas (areas with copula value less than or equal to a given q). Though an analytical expression of K_C might not exist for non-Archimedean copulas, it can be numerically constructed from Monte Carlo simulations, and then the empirical Kendall distribution function K_C may be constructed as (example shown in Kao and Govindaraju, 2008):

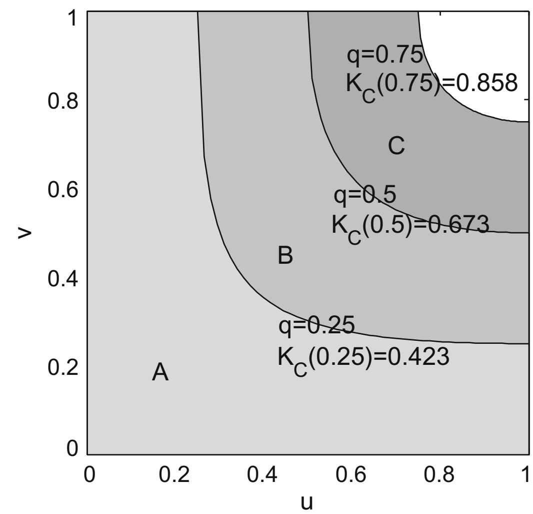

Fig. 1. Distribution function K_C of Gumbel-Hougaard Archimedean copulas with dependence parameter $\theta = 2$. K_C provides the probability measure for the shaded region where $C(u, v) \leqslant q$ ($K_C(0.25)$ for region A, $K_C(0.5)$ for regions A + B, and $K_C(0.75)$ for regions A + B + C).

$$K_{C_n}\left(\frac{l}{n}\right) = \frac{b}{n} \tag{4}$$

where b is the number of samples $\{x_1, \ldots, x_d\}$ with $C_n(k_1/n, \ldots, k_d/n) \leqslant l/n$. Of particular interest is that K_C allows us to compute the probabilistic measure of the joint deficit status, which can be further translated to a joint drought index.

Construction of the copula-based joint drought deficit index

In this study, the standardized index (SI) introduced by McKee et al. (1993) is adopted for statistical analyses of hydrologic variables (when applied to precipitation, it is the well-known SPI). There are several reasons for adopting SI: (1) it can be applied to precipitation, streamflow, and other variables; (2) it does not incur model assumptions that are typical for other indices such as PDSI; (3) it is a probability measure of cumulative precipitation by definition. Therefore, drought severity is scaled in terms of probabilities that can be compared between various locations and among variables. The procedure for constructing joint distributions includes: (1) identifying marginal distributions; (2) selecting suitable dependence structure (copulas); and (3) forming joint distributions. In the following section, the marginals of joint deficit distribution will be characterized using SI method. Since the current SI approach cannot account for the seasonal variability, an amendment is also proposed in order to obtain a more statistically sound SI.

Marginal distribution

Taking rainfall as an example, let $D(t)$ represent the rainfall depth measured at time t ($\Delta t = 1$ month in this study) and the aggregated values $X_w(t) = \sum_{i=t-w+1}^{t} D(i)$ indicate the total precipitation for a given w-month window with respect to t. By analyzing the whole series of X_w for each w, the marginal CDF $u_w = F_{X_w}(x_w)$ is obtained by fitting a 2-parameter Gamma (G2) distribution as suggested by McKee et al. (1993). Then the index SI_w is computed by taking the inverse normal $\phi^{-1}(u_w)$, i.e.

$$SI_w = \phi^{-1}(u_w) = \phi^{-1}(F_{X_w}(x_w)) \quad (5)$$

Therefore, a positive SI_w indicates wet conditions ($0.5 < u_w < 1$), a negative SI_w indicates dry conditions ($0 < u_w < 0.5$), and $SI_w = 0$ indicates normal conditions (median, $u_w = 0.5$) for the w-month window. In other words, the precipitation aggregates $X_w(t)$ are transformed to dimensionless indices SI_w and hence drought severities under various temporal windows w at time t can be compared directly. This method can be extended to other types of hydrologic time series as well. For streamflow data, let $R(t)$ represent the monthly discharge at time t, then the mean discharge within a given w-month window can be defined as $Y_w(t) = \sum_{i=t-w+1}^{t} R(i)/w$. By identifying a suitable probability function for Y_w, the SI_w for streamflow can also be constructed from CDFs $v_w = F_{Y_w}(y_w)$. Referring to the definition of drought categories used in US Drought Monitor (Svoboda et al., 2002), Table 1 shows the range of SI values along with their probabilities of occurrence and corresponding drought conditions.

While this procedure seems intuitively reasonable, it has several flaws. For instance, significant auto-correlation may exist in the samples (note that different X_w may overlap each other) and cause the fitting of probability distributions to be biased. For instance, X_3 of February, 1999 has 2 months in common with X_3 of January, 1999 (rainfall depths D of December 1998 and January 1999), but they are still merged together with all other X_3s to derive F_{X_3}. This problem will become more severe for larger w, because for larger windows the samples will overlap more. The other concern is of seasonal variability. Fig. 2a shows an example of mean monthly precipitation along with the corresponding standard deviation for COOP station Alpine 2 NE (COOPID: 120132), and Fig. 2b shows mean monthly discharge and standard deviation for the nearby streamflow station Whitewater River (USGS Site 03275000). The precipitation data from station Alpine 2 NE has been combined with data from two nearby stations Mauzy (COOPID: 125050) and Liberty 3 SSE (COOPID: 125435) to ensure continuous measurement from 1893 to 2006 (114 years), and the station Whitewater River has discharge data recorded from 1929 to 2006 (78 years). For precipitation, February is the driest month and May the wettest, while for streamflow April is the wettest and September the driest. Similar observations were made for other stations in Indiana as well. From Fig. 2, it can be seen that strong seasonal patterns exist both in precipitation and streamflow. Therefore, a given amount of hydrologic quantity reflects different moisture deficit status depending on when it is observed (e.g., 75-mm monthly rainfall in May is less than average while the same amount is more than average in February). However, this seasonal difference cannot be reflected in the conventional SI approach.

To resolve the above issues, one can further group X_w by its ending month (month of $D(t)$) to form subsets X_w^{month} s, where *month = Jan, Feb, ..., Dec*. In other words, the series $X_w(t)$ is subdivided into 12 smaller series, i.e. $X_w^{\text{month}}(g) = X_w(12(g-1)+m) = X_w(t)$, in which $g = 1, 2, \ldots$, is the year index, $m = 1$ (*Jan*), 2 (*Feb*), ..., 12 (*Dec*) is the month index, and the time index $t = 12(g-1) + m$. Therefore, X_1^{Jan} represents January precipitation, and X_6^{Aug} represents the 6-month precipitation total from March to August. In doing so, samples in each X_w^{month} set are collected annually and will be non-overlapping when $w \leqslant 12$ (note that Δt is 1 month while Δg is 1 year). In other words, the degree of auto-correlation among samples will be largely reduced. Besides, samples within the same group X_w^{month} are subject to the same seasonal effect (i.e. spanning for the same months of the year), and hence the seasonal variation is accounted for in an appropriate manner. This is conceptually similar to the approach used to generate weekly residuals in Kuhn et al. (2007). By fitting distributions separately for each group (i.e., constructing $u_w^{Jan} = F_{X_w^{Jan}}(x_w^{Jan})$, $u_w^{Feb} = F_{X_w^{Feb}}(x_w^{Feb}), \ldots$, and $u_w^{Dec} = F_{X_w^{Dec}}(x_w^{Dec}), w = 1, 2, \ldots$), the modified SI_w^{mod} can be computed similar to SI_w by:

$$SI_w^{\text{mod}} = \phi^{-1}(u_w^{\text{month}}) = \phi^{-1}(F_{X_w^{\text{month}}}(x_w^{\text{month}})) \quad (6)$$

Similarly, this modified approach can be applied on streamflow data as well, in which the variables $Y_w^{\text{month}}(g) = Y_w(m + 12(g-1)) = Y_w(t)$ and $v_w^{\text{month}} = F_{Y_w^{\text{month}}}(y_w^{\text{month}})$ are defined analogously.

Dependence structure

The SI values described above give a standardized moisture abnormality measure within a w-month window at time t. In order to assess the overall drought status, all abnormalities under various w-month windows need to be examined together. Since droughts are slowly evolving phenomena, strong temporal auto-correlation among SIs is expected. Instead of using conventional time series analysis methods (e.g. PDSI is an AR(1) model), copulas were utilized in this study to model their temporal dependence structure. In order to capture both short- and medium-term droughts, window sizes from 1-, 2- to 12-month ($w = 1, 2, \ldots, 12$) are selected, i.e. the set $\{SI_1, SI_2, \ldots, SI_{12}\}$ is of interest. Since the seasonal variability has been corrected via the modified SI at the marginal level as evidenced by Table 2, it is assumed that the dependence structure of standardized indices $\{SI_1^{\text{mod}}, SI_2^{\text{mod}}, \ldots, SI_{12}^{\text{mod}}\}$ will not change significantly with season.

The precipitation marginals u_w^{mod} and streamflow marginals v_w^{mod} for a given w-month window are given by the corresponding $\phi(SI_w^{\text{mod}})$ values. We note that u_w^{mod} and v_w^{mod} are unions of u_w^{month} and v_w^{month} for each month. Marginals $\{u_1^{\text{mod}}, u_2^{\text{mod}}, \ldots, u_{12}^{\text{mod}}\}$ and $\{v_1^{\text{mod}}, v_2^{\text{mod}}, \ldots, v_{12}^{\text{mod}}\}$ with window sizes w from 1- to 12-month with respect to the same ending time were constructed (e.g., u_1^{mod} is the union of $u_1^{Jan}, u_1^{Feb}, \ldots, u_1^{Dec}$). The choice of $\{u_1^{\text{mod}}, u_2^{\text{mod}}, \ldots, u_{12}^{\text{mod}}\}$ and $\{v_1^{\text{mod}}, v_2^{\text{mod}}, \ldots, v_{12}^{\text{mod}}\}$ in forming high

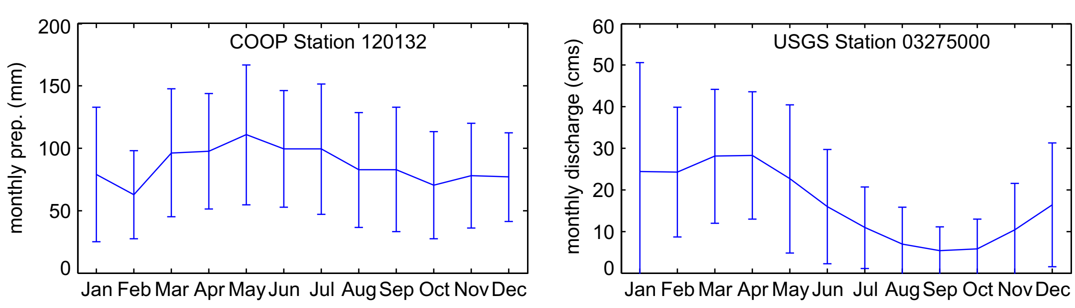

Fig. 2. The monthly average and standard deviation of: (a) left: precipitation station Alpine 2 NE (COOPID: 120132) and (b) right: streamflow station Whitewater River (USGS Site 03275000).

Table 2
Summary of the observed frequencies of droughts detected by both conventional and modified 1-month SI from 73 precipitation and 36 streamflow stations in Indiana. Droughts are defined as SI values less than −0.52 (including categories D0–D4).

Month	January	February	March	April	May	June	July	August	September	October	November	December
Precipitation droughts												
Rank of mean monthly precipitation (low to high)	2	1	8	9	12	11	10	7	6	3	5	4
Drought frequency reported by SI_1 (%)	41.2	48.6	21.2	14.4	13.3	15.3	19.4	22.5	30.5	35.6	26.4	31.8
Drought frequency reported by SI_1^{mod} (%)	28.9	25.9	26.9	28.0	27.5	27.8	26.6	27.8	25.0	26.4	26.8	26.8
Streamflow droughts												
Rank of mean monthly discharge (low to high)	8	10	12	11	9	6	4	3	1	2	5	7
Drought frequency reported by SI_1 (%)	22.3	12.9	2.6	3.0	9.5	25.8	50.2	75.0	79.4	74.0	48.0	31.8
Drought frequency reported by SI_1^{mod} (%)	31.4	33.6	31.2	32.0	34.5	32.0	32.1	31.6	32.9	33.6	36.6	35.6

dimensional copulas increases the complexity of the dependence model. Nevertheless, it is required because drought durations exhibit wide temporal variations, and only by encompassing various durations (from 1- to 12-month) can one represent droughts. An annual cycle accounts for seasonal effects naturally. Moreover, this construct allows for a month-by-month assessment for future conditions, as will be shown later. Marginals longer than 12-month ($j > 12$) are excluded from this study because the adopted samples will start to overlap each other (samples no longer independent) and cause the fitting result to be biased even after applying the modified SI procedure presented in the previous section.

Definition of joint deficit index

Since droughts do not have fixed temporal scales, multiple SIs with various window sizes need to be examined together to judge the overall status of a drought. Therefore, we propose to adopt the joint cumulative probability of SIs as the overall drought indicator in this study.

For temporal windows of interest (say $\{u_1^{mod}, u_2^{mod}, \ldots, u_{12}^{mod}\}$ covering 1–12-month), a copula gives cumulative probability $P[U_1 \le u_1^{mod}, \ldots, U_{12} \le u_{12}^{mod}] = q$. Clearly, when a drought occurs, most of the marginals u_w^{mod} will be small and result in a small value of q. On the other hand, when a drought is not serious, moisture deficits should be higher and result in a higher q. The cumulative probability q can be viewed as the *joint deficit status* of given marginal sets (e.g., $\{u_1^{mod}, u_2^{mod}, \ldots, u_{12}^{mod}\}$), where a smaller q implies overall drought conditions while a larger q implies overall wet conditions. This joint deficit status q is determined through probabilistic considerations, and offers an objective measure of droughts.

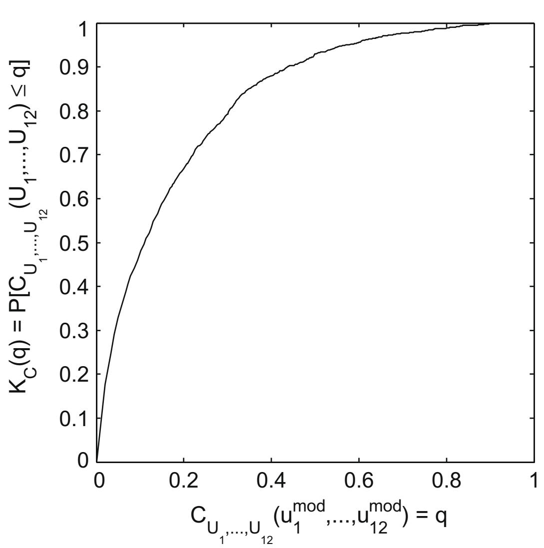

Fig. 3. Kendall distribution function $K_C(q) = P[C_{U_1,\ldots,U_{12}}(U_1^{mod}, \ldots, U_{12}^{mod}) \le q]$ of copulas $C_{U_1,\ldots,U_{12}}(u_1^{mod}, \ldots, u_{12}^{mod})$ of precipitation station Alpine 2 NE (COOPID: 120132).

We note that the joint deficit status q is linked to the given set of marginals and is only comparable to other q with the same set of marginals. For instance, q_1 for $\{u_1^{mod}, u_2^{mod}, \ldots, u_{12}^{mod}\}$ and q_2 for $\{u_1^{mod}, u_2^{mod}, \ldots, u_{11}^{mod}\}$ are not comparable. Therefore, a more general index based on q is desirable. By assuming that events with the same value q will have similar joint drought severity, it will be of interest to know the cumulative probability for events with joint deficit status less than or equal to a given threshold q (i.e., $P[C_{U_1,\ldots,U_{12}}(U_1, \ldots, U_{12}) \le q]$). The Kendall distribution function K_C provides this probability. An example of $K_C(t)$ using precipitation marginals $\{u_1^{mod}, u_2^{mod}, \ldots, u_{12}^{mod}\}$ of station Alpine 2 NE is shown in Fig. 3. In this example, the probability of events with copula value q less than 0.3 is about 0.8. Following the discussion in "Marginal distributions for modified monthly SI values", a *joint deficit index* (JDI) can be defined analogously to SI:

$$\mathrm{JDI} = \phi^{-1}(K_C(q)) = \phi^{-1}(P[C_{U_1,\ldots,U_{12}}(U_1, \ldots, U_{12}) \le q]) \tag{7}$$

where positive JDI ($0.5 < K_C < 1$) implies overall wet conditions, negative JDI ($0 < K_C < 0.5$) indicates overall dry conditions, and JDI zero ($K_C = 0$) indicates normal conditions. In other words, JDI is based on the cumulative probability of joint deficit status q. A very extreme drought will result in a small q, and the JDI will correspondingly yield a low probability.

Data used in this study

The focus of this study is on the State of Indiana. In order to construct reliable multivariate statistical models of joint drought deficit distributions, large amounts of sufficiently long historic observations are desirable. For instance, a 50-year minimum recording length is adopted by National Weather Service (NWS) in performing at-site rainfall frequency analysis (Bonnin et al., 2004). This 50-year standard was also chosen in this study as a minimum requirement. When longer records are available for some variables, this standard can be further raised to increase reliability. Considering the nature of droughts, stations were considered acceptable if monthly precipitation data had an 80-year minimum recording length, and monthly streamflow data contained 50-years minimum recording length.

Precipitation records were obtained from the daily surface dataset (TD 3200) of cooperative stations (COOP) from National Climate Data Center (NCDC). After data processing (such as combination of nearby stations), a total number of 73 stations with record lengths greater than 80 years were obtained. Monthly precipitation was computed based on aggregated daily values. In cases where data were missing for the entire month (around 1.22% of the entire precipitation dataset), they were replaced by the historic mean of that specific month (i.e., assuming moisture status of that unknown month to be neither wet nor dry).

For streamflow, the United States Geological Survey (USGS) daily streamflow dataset was utilized in the present study. Unlike precipitation, streamflow data are subjected to human interference, and therefore data contain both natural and regulated flows. This practice does not cause serious problems in flood frequency analysis (flows are much larger), but it is expected to result in more errors for low flow (drought) conditions. Therefore, only unregulated stations were included to ensure an unbiased analysis. After imposing the 50-year record length requirement, a total of 36 unregulated stations were available for the study area. Daily mean flow data were collected and processed to form monthly mean discharges. The missing values were interpolated in the same way as precipitation and 1.38% of the data were replaced by historic mean monthly streamflows. We adopted copulas for conducting primarily at-site analyses to capture temporal evolution of droughts, and use spatial interpolation for assessing the behavior over the entire state.

Results and applications

Role of seasonality

We first demonstrate the role of seasonality by examining the 1954 precipitation data for station Alpine 2 NE as an example. In Fig. 4a, monthly precipitation (i.e., both X_1s and X_1^{month} s) of year-1954 are shown. It can be seen that similar amounts of precipitation (around 50-mm) are observed in February, March, and May, hence values of conventional SI_1 are close to each other for these 3 months as shown in Fig. 4b. Nevertheless, since it is wettest in May and driest in February (shown in Fig. 2), a 50-mm rainfall should not have the same implication in May as in February. With the modified approach, this seasonal variability is accounted for as evidenced by the higher $\mathrm{SI}_1^{\mathrm{mod}}$ of February and lower $\mathrm{SI}_1^{\mathrm{mod}}$ of May in Fig. 4c.

A further examination, using the entire precipitation and streamflow datasets of Indiana, is shown in Table 2. The numbers of 1-month droughts detected by SI_1 and $\mathrm{SI}_1^{\mathrm{mod}}$ when their values are less than −0.52 (i.e., including all drought categories D0∼D4) are computed and transformed to observed frequencies. The ranks (low to high) of both average monthly rainfall depth and mean streamflow discharge are also included. Columns of the driest and wettest months are marked in gray for illustration. For the conventional SI approach, the highest observed frequencies are found in the driest months (48.6% in February months for precipitation droughts, and 79.4% in September months for streamflow droughts), while the lowest frequencies were found in the wettest months (only 13.3% in May months for precipitation droughts and 2.6% in March months for streamflow droughts). As for the modified approach, the observed drought frequencies were found to be around 30% corresponding to the drought definition given in Table 1. This result is expected since SI_1 is normalized by all 1-month data without considering the seasonal variability. In order words, the moisture abnormalities reported by modified SI corresponds to the monthly mean while the conventional SI to the overall mean. Since most human and agricultural activities have adapted to the local seasonal pattern, a suitable drought index should aim at reporting abnormalities in specific seasons or months instead of reflecting the known seasonal behavior. For locations with larger seasonal variability than Indiana, the performance of the conventional SI as demonstrated in Table 2 is likely to be more dramatic. In addition, the difference between SI_w and $\mathrm{SI}_w^{\mathrm{mod}}$ is largest for $w = 1$, and least for $w = 12$ when seasonal variability is no longer a controlling factor.

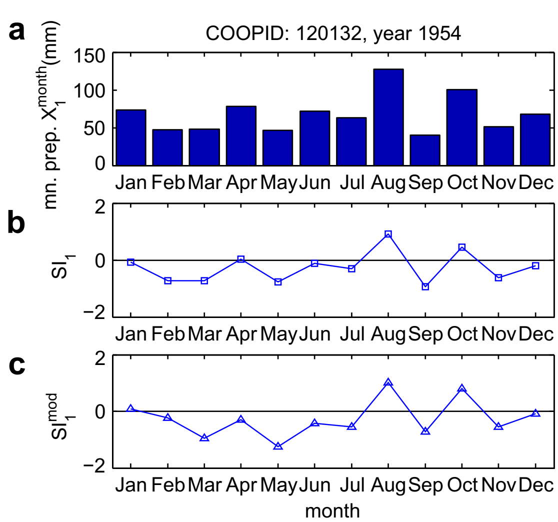

Fig. 4. Comparison between conventional and modified 1-month SI using precipitation station Alpine 2 NE (COOPID: 120132): (a) upper panel: monthly precipitation of year 1954 (X_1^{month} series); (b) middle panel: conventional SI_1 of each month; (c) lower panel: modified $\mathrm{SI}_1^{\mathrm{mod}}$ of each month.

Marginal distributions for modified monthly SI values

While the G2 distribution was suggested by McKee et al. (1993) for computing conventional SPI, it may not be suitable for the modified SI for rainfall or streamflow. In order to test the appropriateness of G2, Kolmogorov–Smirnov (KS) and Cramer-von Mises (CM) tests were applied for the goodness of fit at the 5% significance level in this study, with model parameters estimated by maximum likelihood (ML) method (see Rao and Hamed (2000) and Laio (2004) for mathematical details). For the 73 selected precipitation stations in Indiana, G2 was fitted to both conventional (X_w) and modified (X_w^{month}) sets of cumulated precipitation with window sizes ranging from 1- to 12-month ($w = 1, 2, \ldots, 12$), and the test results are reported in Table 3. It was found that 142 and 99 out of 876 cases (73 stations with 12 window sizes) failed to pass KS and CM tests, respectively, with conventional definitions of SI. With the modified SI definition, only 122 and 117 out of 10,512 cases (73 stations, 12 window sizes, and 12 ending months) failed to pass these two tests, respectively. Therefore, G2 was deemed to be suitable for the proposed approach in computing modified SI of precipitation. For streamflow, a total of 432 cases (36 stations with 12 window sizes) were tested for G2 by conventional approach and 5184 cases (36 stations, 12 window sizes, and 12 ending months) by the modified approach. Note that the modified SI has 12 times more cases to account for seasonality. It was found that G2 was not suitable (a very large percentage of failing cases was suggested by CM test), and instead the generalized extreme value (GEV) distribution was tested for streamflows. As shown in Table 3, GEV provides appropriate fits for most of the cases in computing modified SI of streamflow, and was thus adopted for further analysis.

Table 3
Summary of goodness-of-fit tests on SI. Numbers of cases that are rejected by Kolmogorov–Smirnov (KS) and Cramer-von Mises (CM) tests at the 5% significant level from a total of 73 precipitation stations and 36 streamflow stations are reported. The window sizes used in both SI and modified SI are from 1- to 12-month.

	Precipitation	Streamflow	
	G2	G2	GEV
KS test			
SI_w	142/876	287/432	163/432
$\mathrm{SI}_w^{\mathrm{mod}}$	122/10,512	190/5184	11/5184
CM test			
SI_w	99/876	415/432	144/432
$\mathrm{SI}_w^{\mathrm{mod}}$	117/10,512	4161/5184	0/5184

Though the modified SI approach helps account for seasonality, it requires a longer recording length in order to yield reliable marginal distributions.

Dependence structure specification

For an assessment of the nature of dependence, Spearman's rank correlation coefficient $r_{i,j}$ between each pair of $\{u_1^{\text{mod}}, u_2^{\text{mod}}, \ldots, u_{12}^{\text{mod}}\}$ from precipitation station Alpine 2 NE and $\{v_1^{\text{mod}}, v_2^{\text{mod}}, \ldots, v_{12}^{\text{mod}}\}$ from streamflow station Whitewater River were computed and reported in Table 4 as an example. While $r_{i,j}$ in Table 4 were computed using marginals u_w^{mod} and v_w^{mod}, they can be further interpreted as the rank correlation coefficient between SI_w^{mod} with various window sizes since SI_w^{mod} are monotonically increasing transformation of marginals (i.e., ranks remain the same).

For precipitation marginals (upper triangle in Table 4), it can be seen that short-term marginal u_1^{mod} has a high correlation of 0.71 with u_2^{mod}, and the correlation decays quite fast with increasing window size j (when $j \geqslant 6$, $r_{1,j}$ between u_1^{mod} and u_j^{mod} are less than 0.4). For a long-term marginal u_{12}^{mod}, it has a high correlation with u_j^{mod} for $j \geqslant 4$, and the correlation becomes less for short-term windows. While u_1^{mod} represents the past month precipitation status (important for identifying emerging droughts) and u_{12}^{mod} represents the past year precipitation status (important for identifying prolonged droughts), they are not well-correlated to each other and hence none of them can be overlooked. Table 4 also shows that no representative window exists, i.e. no single u_i^{mod} can solely represent others (highly dependent on every other u_j^{mod}), and hence every single u_i^{mod} can only reflect a partial view of a precipitation drought. A similar observation can be made from streamflow marginals (lower triangle in Table 4). Comparing to the corresponding precipitation marginals, streamflow has a higher level of temporal correlation as expected. Nevertheless, a single window size cannot be used to represent the entire hydrologic drought status.

The joint deficit index (JDI)

Since the JDI is on an inverse normal scale (same as SI), the classifications listed in Table 1 can be adopted for JDI as well. An illustration of JDI is shown in Fig. 5 using precipitation marginals of station Alpine 2 NE. JDI, SI_w^{mod}, $w = 1, 2, \ldots, 12$ and the corresponding 12-month precipitation are presented for four select cases. In Fig. 5a, SI_w^{mod} values observed in all window sizes for June 1988 report serious precipitation deficits indicating a severe drought, which is also suggested by the JDI. Fig. 5b shows an opposite case, in which JDI and SI_w^{mod} observed in all window sizes for August 1985 report sufficient precipitation. One can notice that JDI is slightly higher than all other SI_w^{mod} in this instance. Since JDI is based on the joint probability of all SI_w^{mod}, it suggests that the joint behavior of drought cannot be assessed by a simple weighted average of SI_w^{mod} due to the effect of dependence structure. Fig. 5c shows a case of emerging drought in February 1978, in which SI_1^{mod} is quite small due to the serious precipitation deficit in February while other SI_w^{mod} are above normal. This case would be confusing for interpretation since most of the indices do not capture droughts in a timely manner. The JDI reflects a drought condition based on the entire dependence structure. Fig. 5d shows a prolonged drought in October 1988, in which SI_1^{mod} and SI_2^{mod} report sufficient precipitation in September and October while other SI_w^{mod} report precipitation deficit due to the preceding serious drought (shown in Fig. 5a). This is another instance where the JDI can provide the probability of joint deficit status objectively and reflect drought conditions better than SI.

A regional illustration is shown in Fig. 6, where a comparison between SI_1^{mod}, $\text{SI}_{12}^{\text{mod}}$, and the proposed JDI is performed using Indiana precipitation data for June and July, 1988, which correspond to one of the most serious droughts recorded in history. For June 1988, hardly any precipitation was observed and hence SI_1^{mod} indicates extremely dry conditions. However, $\text{SI}_{12}^{\text{mod}}$ does not show a similar severity because its long-term memory delays the response for an emerging drought. For the following month, approximately normal precipitation was observed but it did not relieve the accumulated deficit as suggested by the $\text{SI}_{12}^{\text{mod}}$. However, SI_1^{mod} falsely indicates that the deficit status has come back to normal because it lacks longer memory. As for the proposed JDI, it is found to better reflect the emerging drought (in June), and also has temporal memory for accumulated deficit (in July).

The most important feature of JDI is that the overall deficit status is based on the dependence structure of deficit indices with various temporal windows. When the deficit indices are found to be uniformly low, the resulting joint drought index will be

Table 4
Spearman's rank correlation coefficient r_{ij} between precipitation marginals u_i^{mod} and u_j^{mod} (upper triangle) and streamflow marginals v_i^{mod} and v_j^{mod} (lower triangle) of precipitation station Alpine 2 NE and streamflow station Whitewater River (note that $r_{ij} = r_{ji}$).

Spearman's $r_{i,j}$ between v_i^{mod} and v_j^{mod}	j \ i	Spearman's $r_{i,j}$ between u_i^{mod} and u_j^{mod}: 1	2	3	4	5	6	7	8	9	10	11	12
	1		0.71	0.57	0.48	0.41	0.38	0.37	0.36	0.35	0.33	0.31	0.30
	2	0.89		0.82	0.70	0.61	0.55	0.53	0.51	0.49	0.47	0.44	0.42
	3	0.80	0.93		0.87	0.76	0.69	0.64	0.61	0.59	0.56	0.54	0.51
	4	0.73	0.85	0.94		0.90	0.81	0.75	0.70	0.67	0.65	0.62	0.60
	5	0.67	0.78	0.87	0.95		0.92	0.85	0.79	0.75	0.72	0.69	0.67
	6	0.63	0.72	0.81	0.89	0.96		0.93	0.87	0.82	0.78	0.75	0.73
	7	0.59	0.68	0.75	0.83	0.90	0.96		0.94	0.89	0.85	0.81	0.78
	8	0.57	0.64	0.72	0.79	0.85	0.91	0.97		0.95	0.90	0.86	0.83
	9	0.55	0.62	0.69	0.75	0.81	0.87	0.93	0.97		0.96	0.91	0.88
	10	0.53	0.60	0.66	0.72	0.78	0.83	0.89	0.94	0.98		0.96	0.92
	11	0.51	0.58	0.64	0.70	0.75	0.81	0.85	0.90	0.94	0.98		0.96
	12	0.50	0.56	0.62	0.68	0.73	0.78	0.83	0.87	0.91	0.95	0.98	

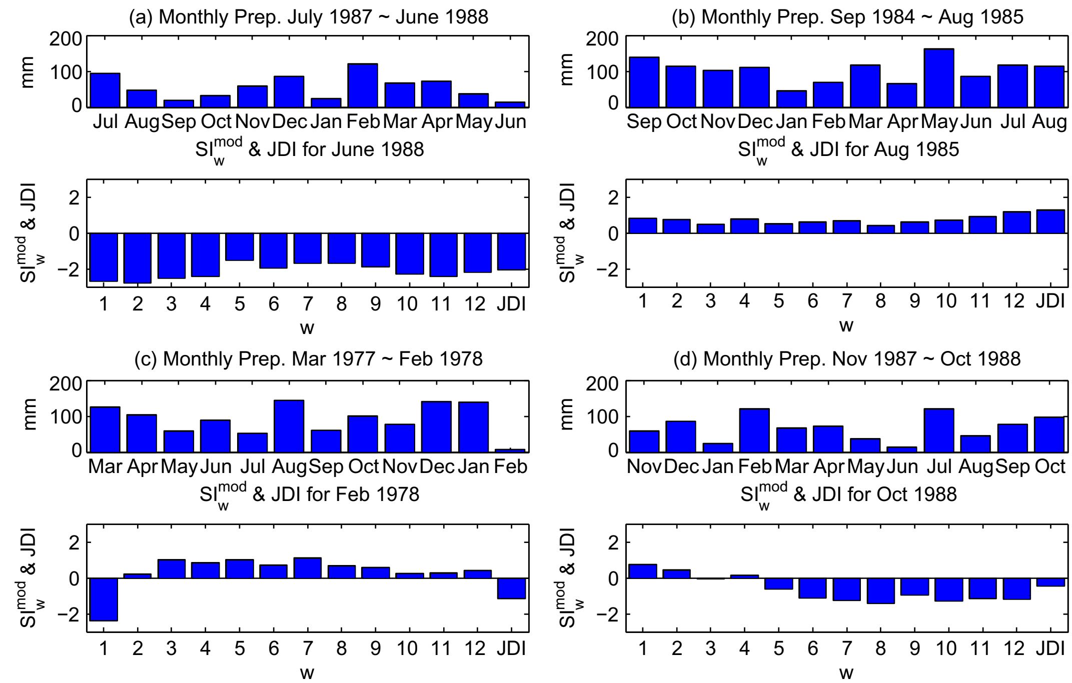

Fig. 5. Illustration of JDI and SI_w^{mod}, $w = 1, 2, \ldots, 12$ of precipitation station Alpine 2 NE (COOPID: 120132) for four selected cases.

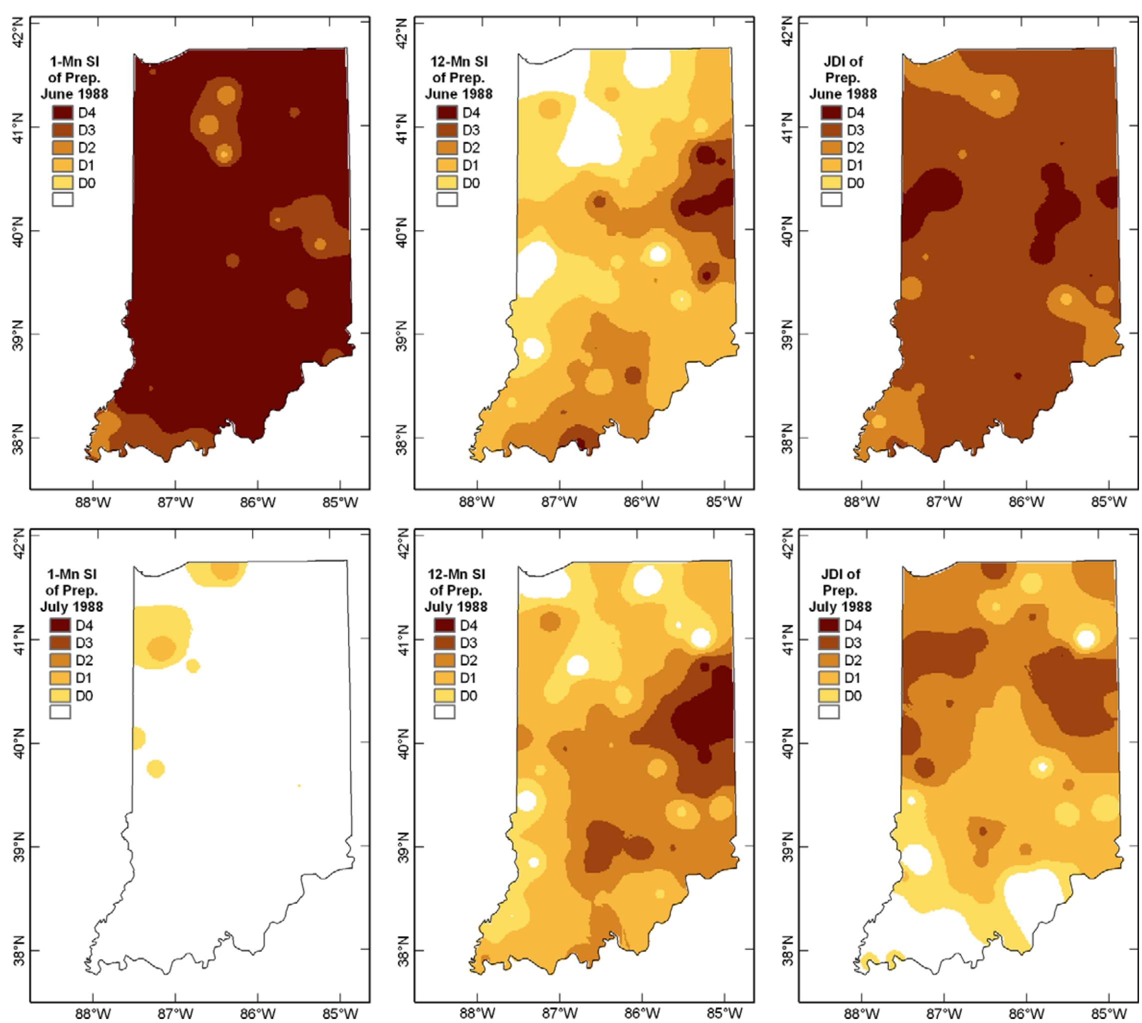

Fig. 6. Regional illustration of SI_1^{mod} (left), SI_{12}^{mod} (middle), and JDI (right) of precipitation of Indiana for June and July 1988.

extremized due to its rareness. Though the examples shown in Figs. 5 and 6 are based on precipitation deficit, JDI can be constructed using other hydrologic quantities as well. A comparison between precipitation and streamflow droughts will be performed using JDI in the following section.

Comparison between precipitation and streamflow deficits using JDI

Though droughts are fundamentally triggered by insufficient precipitation, the evolution of a drought from precipitation to streamflow is not instantaneous and is controlled by complex physical mechanisms. To study the correlation between these two different types of deficits, a general method is required where the analyses can be performed in a systematic manner. The proposed copula approach serves this need because both SI and JDI are not limited to precipitation data alone and are applicable to other hydrologic variables.

Similar to Table 4, the cross correlation between precipitation marginals $\{u_i^{\text{mod}}|i = 1, 2, \ldots, 12\}$ of station Alpine 2 NE and streamflow marginals $\{v_j^{\text{mod}}|j = 1, 2, \ldots, 12\}$ of nearby streamflow station Whitewater River are examined using Spearman's $r_{i,j}$ and reported in Table 5. It is observed that the correlation $r_{i,i}$ (i.e., $r_{1,1}, \ldots, r_{12,12}$, both marginals have the same window size) increases with increasing window sizes (from $r_{1,1} = 0.62$ to $r_{12,12} = 0.78$). This is expected because as the window size increases, the time lag from precipitation to streamflow is absorbed by the time window. As for $r_{i,1}$ (1-month streamflow v_1^{mod} to various durations of precipitation u_i^{mod}), the correlation remains at a high level between 0.6 and 0.7. This likely reflects the temporal delay from precipitation to streamflow (mechanisms such as baseflow), and hence supports that streamflow deficits have longer temporal memory than precipitation deficits. On the other hand, $r_{1,j}$ (1-month precipitation u_1^{mod} to various lengths of streamflow v_j^{mod}) decays quickly as expected, since past streamflows have little influence on future precipitation.

Following the procedures developed in "Definition of joint deficit index", JDI was computed both for precipitation and streamflow marginals. Furthermore, JDI was constructed for the mixed marginals $\{u_1^{\text{mod}}, \ldots, u_{12}^{\text{mod}}, v_1^{\text{mod}}, \ldots, v_{12}^{\text{mod}}\}$ as well to represent the inter-variable drought information based on the entire dependence structure (both Tables 4 and 5). An example is illustrated in Fig. 7 using precipitation station Alpine 2 NE and streamflow station Whitewater River. It is intriguing to see that both JDIs of precipitation and streamflow are correlated to each other in most of the regions (Spearman's $r = 0.73$ between these two series). Since the developments of these two series are independent of each other, it indicates that similar drought information can be obtained from these two hydrologic variables. This reinforces the claim that the proposed JDI is a general algorithm, and therefore it is a potential approach for analyzing other hydrologic variables as long as the proper marginal distributions and dependence structure can be modeled. It can also be seen from Fig. 7 that the mixed JDI provides intermediate drought severity that is based on the joint distribution of both precipitation and streamflow data at various temporal durations. The regional comparison between precipitation and streamflow JDI is performed in Fig. 8 as an example using Indiana data for April 2001. The general trends of these two JDIs are similar for the illustrated drought event, again demonstrating the viability of JDI for studying time scales of various droughts, and their interdependencies.

Potential of future droughts

As mentioned earlier, the adoption of high dimensional marginals $\{u_1^{\text{mod}}, u_2^{\text{mod}}, \ldots, u_{12}^{\text{mod}}\}$ facilitates a month-by-month future drought potential assessment since JDI is based on temporal dependence structure of deficits. Similar to SI, positive JDI ($K_C > 0.5$) can be interpreted as a joint "wetness" index, negative JDI ($K_C < 0.5$) as the joint dry status, and JDI = 0 ($K_C = 0.5$) represents joint normal status. Therefore, it will be of interest to know under current conditions (e.g. precipitation observations), what amount of precipitation is required in the following months to bring the joint deficit status to normal (JDI = 0 or $K_C = 0.5$). In other words, how much precipitation is required in a future time horizon to recover from an existing drought?

Let P_w^o denote the observed monthly precipitation for the past w months, and P_n^f represent the future n-month monthly precipitation to be assessed. Since the maximum temporal length JDI covered is 12 months, in order to assess P_1^f one needs to have the past 11-month observations $P_1^o, P_2^o, \ldots, P_{11}^o$. Nevertheless, because P_1^f is unknown, the precipitation marginals u_i^{month} cannot be determined directly (e.g., u_1^{month} is controlled by P_1^f, and u_2^{month} is controlled by the sum $(P_1^f + P_1^o)$). In order to solve for the value of P_1^f that results in JDI = 0, one may follow the procedure listed below:

1. Assign an initial guess of P_1^f.
2. Compute precipitation marginals u_1^{month} by P_1^f, u_2^{month} by $(P_1^f + P_1^o)$, …, and u_{12}^{month} by $(P_1^f + \sum_{i=1}^{11} P_i^o)$. The month when P_1^f occurs is the ending month used in the modified SI procedure.
3. Compute $C_{U_1,\ldots,U_{12}}(u_1^{\text{month}}, \ldots, u_{12}^{\text{month}})$, and the corresponding K_C values.
4. Modify P_1^f and repeat 2 and 3 until $K_C = 0.5$.
5. P_1^f will be the required precipitation over the following 1 month in order for the joint deficit status to be normal (i.e. get out of the drought), and $(1 - u_1^{\text{month}})$ will be the probability of this event.

Table 5
Spearman's rank correlation coefficient r_{ij} between precipitation marginals u_i^{mod} and streamflow marginals v_j^{mod}.

	u_1^{mod}	u_2^{mod}	u_3^{mod}	u_4^{mod}	u_5^{mod}	u_6^{mod}	u_7^{mod}	u_8^{mod}	u_9^{mod}	u_{10}^{mod}	u_{11}^{mod}	u_{12}^{mod}
v_1^{mod}	0.62	0.70	0.70	0.69	0.68	0.66	0.65	0.63	0.62	0.61	0.60	0.60
v_2^{mod}	0.42	0.68	0.73	0.75	0.74	0.72	0.70	0.69	0.67	0.66	0.64	0.63
v_3^{mod}	0.32	0.54	0.71	0.76	0.77	0.76	0.74	0.72	0.71	0.69	0.68	0.66
v_4^{mod}	0.25	0.43	0.58	0.73	0.78	0.78	0.77	0.75	0.73	0.72	0.70	0.69
v_5^{mod}	0.21	0.36	0.49	0.63	0.75	0.78	0.78	0.77	0.75	0.73	0.72	0.71
v_6^{mod}	0.20	0.33	0.43	0.55	0.66	0.76	0.78	0.78	0.77	0.76	0.74	0.73
v_7^{mod}	0.17	0.30	0.39	0.49	0.59	0.69	0.76	0.78	0.78	0.77	0.76	0.75
v_8^{mod}	0.18	0.28	0.37	0.46	0.54	0.63	0.70	0.77	0.78	0.78	0.77	0.76
v_9^{mod}	0.17	0.27	0.34	0.43	0.51	0.58	0.65	0.72	0.77	0.78	0.78	0.78
v_{10}^{mod}	0.16	0.26	0.33	0.41	0.48	0.55	0.61	0.67	0.72	0.77	0.78	0.78
v_{11}^{mod}	0.15	0.24	0.32	0.39	0.46	0.52	0.58	0.63	0.68	0.73	0.77	0.78
v_{12}^{mod}	0.15	0.23	0.31	0.38	0.44	0.50	0.55	0.60	0.64	0.69	0.74	0.78

Since all marginals and K_C monotonically increase with increasing P_1^f, this procedure is guaranteed a unique solution. However, required precipitation for other months, such as P_2^f, may not be assessed without assuming a value of prior precipitation P_1^f. Instead of making extra assumptions, the total precipitation $S_n^f = \sum_{i=1}^{n} P_i^f$ in the future n-months ($n \leqslant 12$) may be assessed with the following revised procedure.

1. Assign an initial guess of S_n^f.
2. Compute precipitation marginals u_n^{month} by S_n^f, u_{n+1}^{month} by $(S_n^f + P_1^o), \ldots,$ and u_{12}^{month} by $(S_n^f + \sum_{i=1}^{12-n} P_i^o)$. The month when P_n^f occurs is the ending month used in the modified SI procedure.
3. Compute $C_{U_n,\ldots,U_{12}}(u_n^{\text{month}}, \ldots, u_{12}^{\text{month}})$, and the corresponding K_C values.
4. Modify S_n^f and repeat 2 and 3 until $K_C = 0.5$.

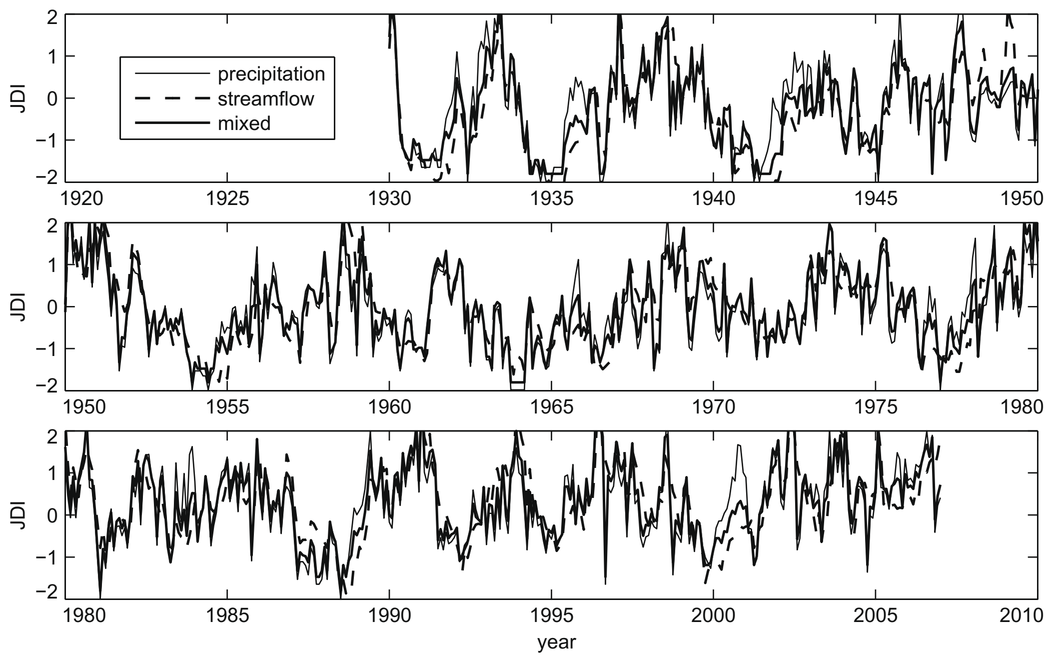

Fig. 7. Comparison of various JDIs constructed by precipitation marginals $\{u_1^{\text{mod}}, u_2^{\text{mod}}, \ldots, u_{12}^{\text{mod}}\}$ of station Alpine 2 NE (COOPID: 120132), streamflow marginals $\{v_1^{\text{mod}}, v_2^{\text{mod}}, \ldots, v_{12}^{\text{mod}}\}$ of station Whitewater River (USGS Site 03275000), and mixed marginals $\{u_1^{\text{mod}}, \ldots, u_{12}^{\text{mod}}, v_1^{\text{mod}}, \ldots, v_{12}^{\text{mod}}\}$.

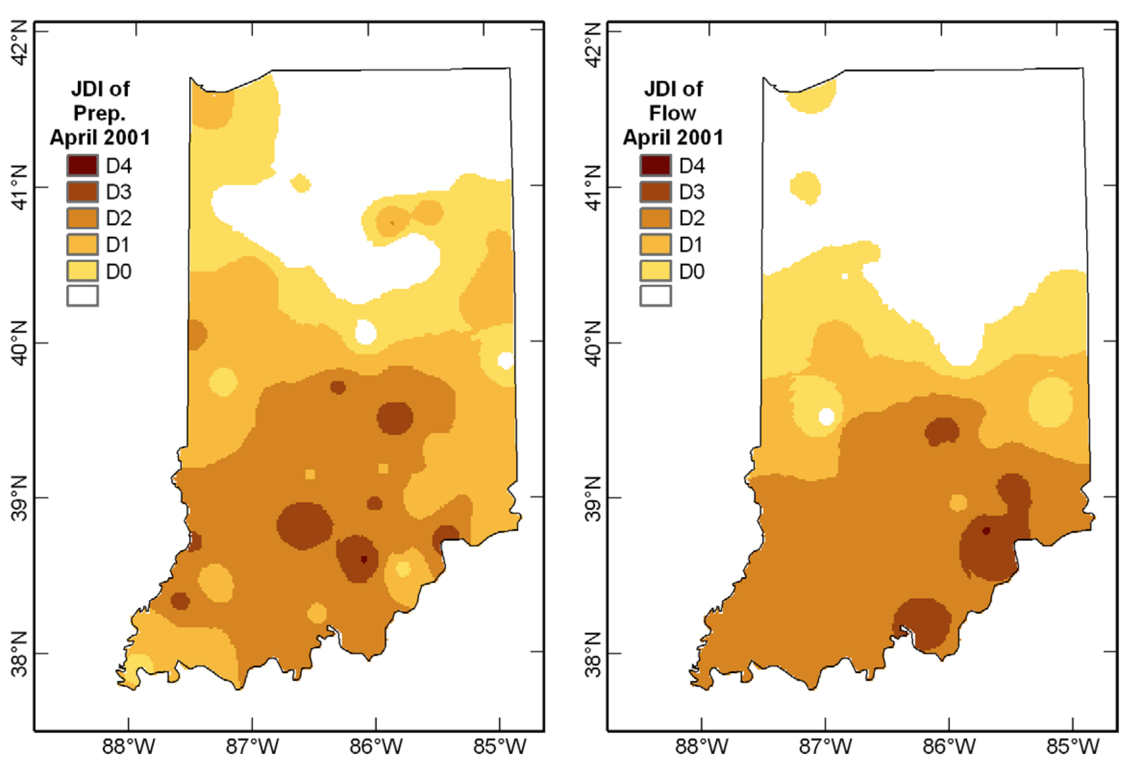

Fig. 8. Regional comparison between precipitation JDI and streamflow JDI of Indiana for April 2001.

5. S_n^f will be the required precipitation for the following n-months in order for the joint deficit status to be normal, and $(1 - u_n^{\text{month}})$ will be the required excess probability.

It should be noted that since we are not willing to make extra assumptions about future precipitation, it is inevitable that a truncated dependence model $C_{U_n,\ldots,U_{12}}(u_n^{\text{month}}, \ldots, u_{12}^{\text{month}})$ needs to be

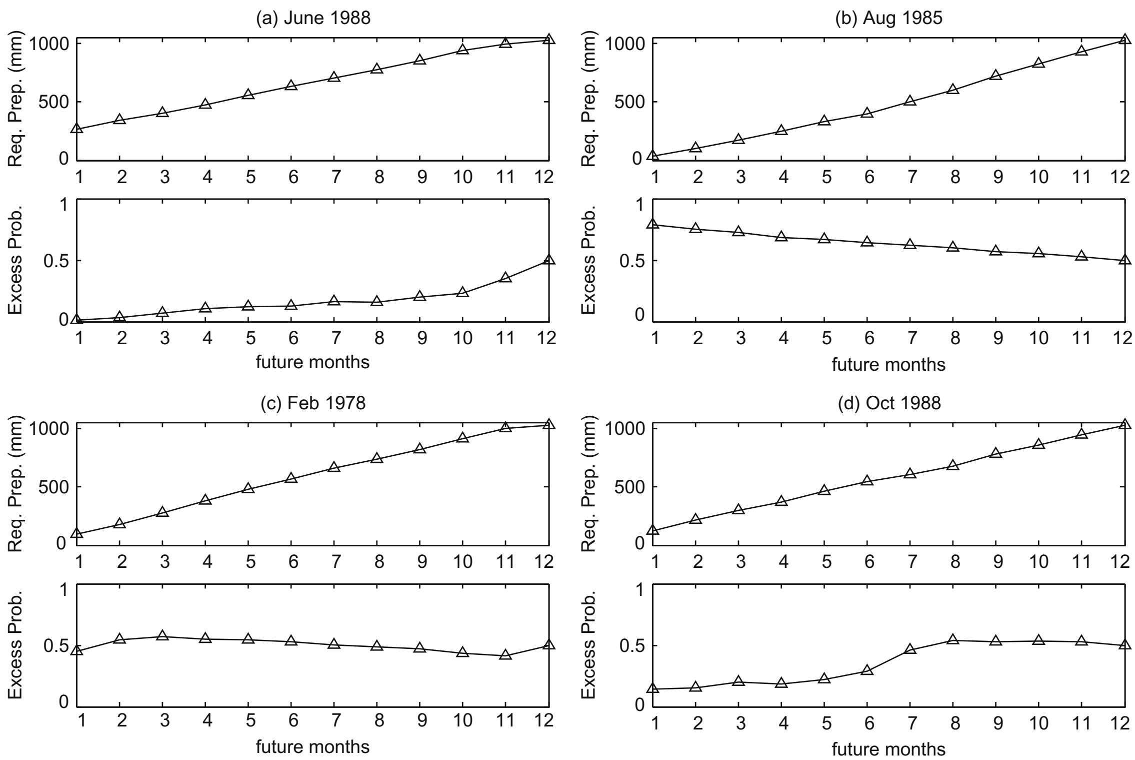

Fig. 9. Required precipitation total in the future months in order to achieve normal status (K_C = 0.5 or JDI = 0), and the corresponding excess probability of station Alpine 2 NE (COOPID: 120132).

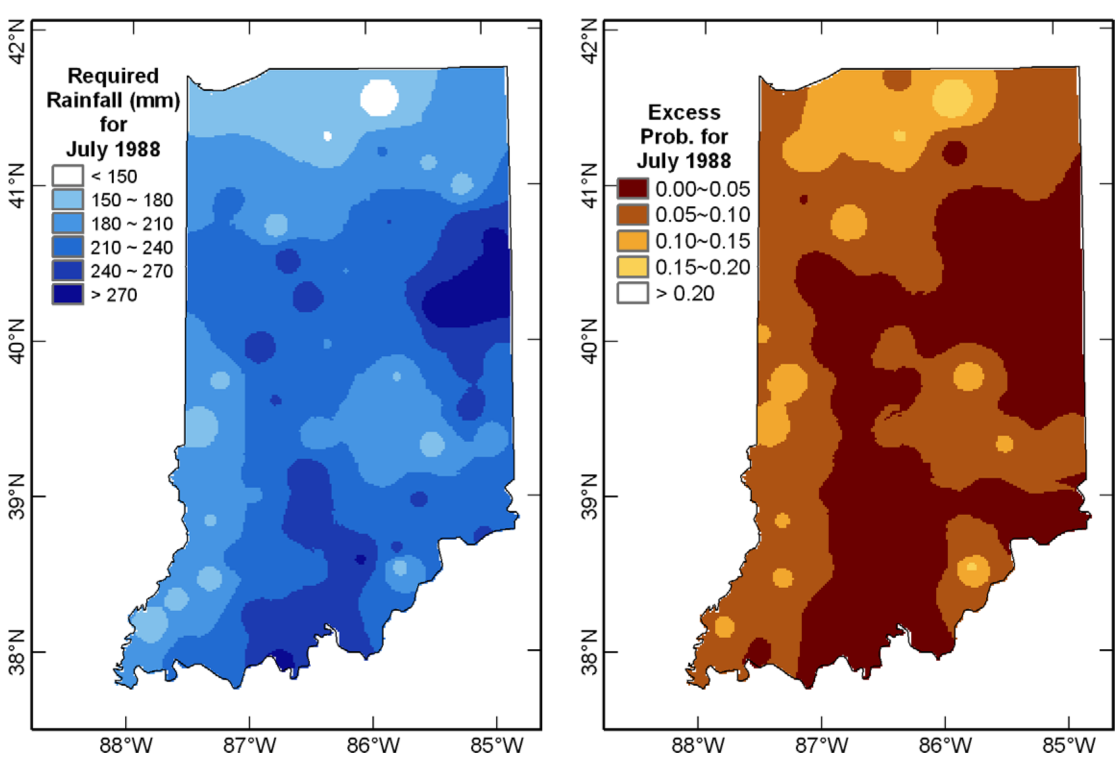

Fig. 10. Regional illustration of (a) left: required precipitation for July 1988 in order to achieve normal status (K_C = 0.5 or JDI = 0) and (b) right: the corresponding excess probability over Indiana based on the observations made from August 1987 to June 1988.

adopted in order to compute the corresponding JDI values. When n reaches 12, copulas $C_{U_d}(u_{12}^{\text{month}})$ and K_C will decay to the marginal u_{12}^{month} since no observed precipitation is available for a conditional estimate. Hence, S_{12}^f corresponds to the median of $F_{X_{12}^{\text{month}}}$ with the ending month same as P_{12}^f.

An example is shown in Fig. 9 using the four selected events adopted in Fig. 5. For Fig. 9a, since the drought is very severe, it requires a very large amount of precipitation (around 250-mm) in order to push JDI back to normal in the following month, which is highly unlikely considering the statistics of monthly precipitation in July. The excess probability provides a clear interpretation of this situation, as it shows that the chance of drought recovery in the following month is nearly zero. For the wet case shown in Fig. 9b, it tells the opposite story as the required precipitation for normal conditions is much less. The excess probability shows that there is a high chance (around 0.8) to remain in wet status for the following month. Fig. 9c is for an emerging drought, where excess probabilities are around median suggesting that though there is serious precipitation deficit in the past 1 month, the future drought potential is less due to the accumulated moisture over the past year. As for the prolonged drought shown in Fig. 9d, it suggests that more than a year's time might be required to recover from drought status despite the rainfall within the past 2 months.

One advantage of the above assessment procedure is that the results are easily understood, as the required precipitation is in units of depth and associated with probability of exceedance. Hence, it can provide useful drought information to interested parties. A regional illustration of the required precipitation for July 1988 to achieve normality based on the observations made from August 1987 to June 1988 and the corresponding probability of exceedance are shown in Fig. 10 (JDI of June 1988 has been shown in Fig. 6). As indicated in Fig. 10, a majority of Indiana would have needed over 150-mm of rain in July 1988. Based on historical July precipitation, this information can be further transformed into exceedance probability, and it suggests similarly that the probability for recovering to normal conditions is quite small (less than 0.1 for most of the state). From historical records, we know that this was not achieved. Such drought maps are a more effective way of relaying drought information, as most of the indices are not amenable to statistical interpretation and are artificially converted into drought severity levels.

Conclusions

The complex relationships between drought-related variables have hindered characterization of overall drought status in the past. However, copulas provide a promising method for characterizing the complicated dependence structure as demonstrated in this study by using a modified SI based on observed rainfall and streamflow data. The following conclusions are presented on the basis of this study.

(1) It was found that the modified SI is not only an index with proper statistical basis, it also helps alleviate the effect of seasonal variability. The gamma distribution was found to be an appropriate distribution for precipitation marginals, while the generalized extreme value distribution was a suitable choice for streamflow marginals based on analysis of Indiana data.
(2) The dependence structures of precipitation and streamflow marginals with window sizes varying from 1- to 12-month were constructed via copulas. Empirical copulas are recommended because of computational efficiency. The reliability of empirical copulas was founded on the large sample sizes adopted in this study.
(3) A joint deficit index (JDI) was proposed in this study using the Kendall distribution function K_C. The JDI can serve as a probability-based drought index from a set of SIs with various temporal window sizes. This is perceived as an improvement over some drought indices that involve subjective judgment or are estimated from linear weighted SIs. Besides providing objective description of the overall drought status, the JDI was shown to be capable of capturing both emerging and prolonged droughts in a timely manner.
(4) The proposed JDI algorithm was found to be potentially suitable for other hydrologic variables, as evidenced by the performance observed between precipitation and streamflow JDIs. The JDI can be further applied on mixed marginals (containing both precipitation and streamflow) to construct an inter-variable drought index, where the entire dependence structure of precipitation and streamflow marginals is preserved.
(5) Drought severity can be assessed through the proposed JDI approach. For instance, the required precipitation for achieving normal conditions (JDI = 0) can be estimated. The required rainfall depth along with its exceedance probability provides good interpretation of drought status.

The utilization of copulas in drought characterization was demonstrated in this study. It is expected that copulas will play an important role in drought analysis and perhaps in other hydrologic studies as well as it enables multidimensional stochastic analysis.

Acknowledgements

Gratitude is expressed to Dr. Venkatesh M. Merwade, Mr. Sanjiv Kumar, Mr. Jonghun Kam, and Miss Kensey Thurner, who organized and provided the use of streamflow data in Indiana. Efforts of the second author were supported in part by the National Science Foundation under Grant No: 0753116. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation. Efforts of the first author were on his own capacity and not on behalf of UT-Battelle, LLC, or its affiliates or successors.

Appendix A:. Introduction to Student *t*-copulas

Student *t*-copulas belong to the family of meta-elliptical copulas (adopted by Genest et al. (2007)), and are an extension of the well-known multivariate Student *t*-distribution (MVT). Student *t*-copulas provide more flexibility in constructing joint distributions where the marginals are not required to be *t*-distributed. The most important feature of Student *t*-copulas is that they are one of the few applicable parametric copula families when the number of variables becomes large (>4), and many existing statistical tools developed for MVT (e.g., Genz and Bretz, 2002) can be applied to Student *t*-copulas as well. Student *t*-copulas can be expressed as:

$$\begin{aligned} C_{U_1,\ldots,U_d}(u_1,\ldots,u_d|\Sigma,\nu) &= \Psi_{d,\nu}(\psi_\nu^{-1}(u_1),\ldots,\psi_\nu^{-1}(u_d)|\Sigma) \\ &= \Psi_{d,\nu}(\psi_\nu^{-1}(F_{X_1}(x_1)),\ldots,\psi_\nu^{-1}(F_{X_d}(x_d))|\Sigma) \end{aligned} \tag{A.1}$$

where ψ_ν is the CDF of the univariate Student *t*-distribution, $\Psi_{d,\nu}$ is the joint-CDF of MVT, and ν is the degrees of freedom. The covariance matrix $\mathbf{\Sigma}$ can be estimated from rank based statistics such as Spearman's correlation coefficient r instead of Pearson's linear correlation coefficient ρ (see Renard and Lang, 2007). Special cases are Cauchy copulas for $\nu = 1$ and Gaussian copulas for $\nu \to \infty$ (both belong to the family of meta-elliptical copulas). The degrees of freedom ν in Student *t*-copulas can be viewed as a parameter that

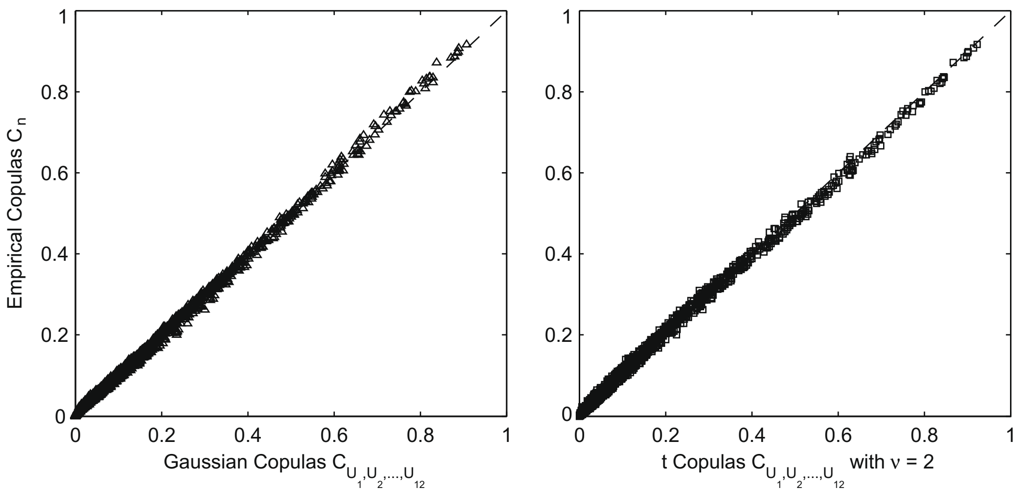

Fig. A.1. Empirical copulas C_n (Eq. (2)) versus (a) left: Gaussian (Eq. (A.1) with $v \to \infty$) and (b) right: Student *t*-copulas (Eq. (A.1) with $v = 2$) for precipitation marginals $\{u_1^{\text{mod}}, u_2^{\text{mod}}, \ldots, u_{12}^{\text{mod}}\}$ of station Alpine 2 NE (COOPID: 120132). The dash line corresponds to the 45-degree line.

allows for modeling fatter tails. Because there is no analytical solution for $\Psi_{d,v}$, one needs to perform a numerical integration:

$$\Psi_{d,v}(\psi_v^{-1}(u_1), \ldots, \psi_v^{-1}(u_d)) = \int_{-\infty}^{\psi_v^{-1}(u_1)} \cdots \int_{-\infty}^{\psi_v^{-1}(u_d)} \frac{\Gamma\left(\frac{p+v}{2}\right)|\Sigma|^{-\frac{1}{2}}}{(\pi v)^{d/2}\Gamma(v/2)} \left(1 + \frac{\mathbf{z}\Sigma^{-1}\mathbf{z}^T}{v}\right)^{-\frac{p+v}{2}} dz_1 \cdots dz_d \tag{A.2}$$

where $\mathbf{z} = [z_1, \ldots, z_d]$ and $\Gamma(\cdot)$ is the gamma function. While this numerical integration can be achieved relatively easily for smaller dimensions (less than or equal to 3 as in Genest et al. (2007)), the computation becomes exceedingly time-consuming for higher dimensions and a Monte Carlo approach is more efficient. An alternative method proposed by Genz and Bretz (2002) is adopted for computing joint-CDFs of MVTs. Through the specification of covariance matrix $\mathbf{\Sigma}$, all bivariate mutual dependencies are preserved in Student *t*-copulas, and hence the complicated dependence structure can be modeled parametrically.

To identify the dependence across temporal scales, one of the statistically exhaustive procedures is to construct the joint distribution of multiple $\mathrm{SI}_w^{\text{mod}}$ values through copulas. Both Student *t* and empirical copulas (Eq. (2)) are adopted to construct dependence structure for the sets $\{u_1^{\text{mod}}, u_2^{\text{mod}}, \ldots, u_{12}^{\text{mod}}\}$ and $\{v_1^{\text{mod}}, v_2^{\text{mod}}, \ldots, v_{12}^{\text{mod}}\}$. For Student *t*-copulas, the covariance matrix $\mathbf{\Sigma}$ is evaluated from Spearman's $r_{i,j}$ using procedures suggested by Phoon et al. (2004), in which each element $\rho_{i,j}$ in $\mathbf{\Sigma}$ is computed by $\rho_{i,j} = 2\sin(\pi r_{i,j}/6)$. Both Renard and Lang (2007) and Genest et al. (2007) provide mathematical details for the use of Gaussian (Student *t* when $v \to \infty$) and general meta-elliptical copulas. In Fig. A.1, empirical copulas are plotted against two cases of Student *t*-copulas, including Gaussian (GAU, Fig. A.1a) and the one with degree of freedom $v = 2$ (T02, Fig. A.1b) as an example using precipitation marginals $\{u_1^{\text{mod}}, u_2^{\text{mod}}, \ldots, u_{12}^{\text{mod}}\}$ of station Alpine 2 NE with data length $n = 1357$ (note that long-term marginal u_{12}^{mod} are not available for the first 11 months of the entire 114-year precipitation data). In contrast to GAU, T02 has a fatter tail and allows for tail dependence (Genest et al., 2007). It can be observed that both these parametric copulas exhibit similar performance when compared to empirical copulas (close to the 45-degree line). By treating the empirical copulas as the observed values, the root mean square error (RMSE) of parametric copulas was computed, and was found to be 0.0083 in Fig. A.1a and 0.0105 in Fig. A.1b. We also checked the higher tail region. There are 61 data points with empirical copula values larger than 0.6, and the RMSE of these points in Fig. A.1a and b are 0.0140 and 0.0160. The results suggested that GAU perform slightly better than T02 for station Alpine 2 NE, but overall their differences are negligible. Similar results are found for other precipitation and streamflow stations as well implying Student *t*-copulas are viable parametric dependence model for droughts. By testing various v values, a most appropriate Student *t*-copula may be identified.

However, the computation of Student *t*-copulas is very demanding. For the 12-dimensional structure, construction of a Student *t*-copula takes several minutes on a personal computer to evaluate one case of Eq. (A.1) and around half a day for a single station adopted in this study. Hence, it is inconvenient for large-scale and multi-site problems such as drought investigation. Therefore, empirical copulas were adopted in the present study. Since the record lengths are quite large (~1400 data points), empirical copulas are reliable as shown in the example in Fig. A.1. We only point out the potential of Student *t*-copulas for drought-related problems. For the study of very extreme droughts (outside the range of C_n) or for truncated models (smaller dimensions), Student *t*-copulas would be a possible choice.

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